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SLIDING MODE CONTROL OF A CLASS OF UNCERTAIN DYNAMIC
SYSTEMS USING LINEAR MATRIX INEQUALITY APPROACH

NOR LAILI BINTI MAHMUD

A project report submitted in partial fulfillment of the
requirements for the award of the degree of
Master of Engineering (Electrical- Mechatronics and Automatic Control)

Faculty of Electrical Engineering
University Technology Malaysia

NOVEMBER 2007

DECLARATION

I declare that this research project report entitled “*Sliding mode control of a class of uncertain dynamic systems using linear matrix inequality approach*” is the result of my own research except as cited in the references. The report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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ABSTRACT

Over the year, many researchers have considered the problem of stabilizing the uncertain systems. In variable structure control (VSC) system, the controller structure around the plant is intentionally changed by using a viable high-speed switching feedback control to obtain a desired plant behavior or response. In this project, the problem of designing a variable structure control (VSC) law for uncertain system is considered. Using LMI approach, a design method of a linear sliding surface which is linear to the state is developed. A sufficient condition for the existence of linear sliding surface is given in term of LMIs and an explicit formula of linear sliding surfaces guaranteeing the quadratic stability of the reduced-order equivalent system dynamics restricted to the sliding surfaces is derived. The sliding mode controller is then applied to an Electrohydraulic servo system. The simulation works were performed using MATLAB/SIMULINK software. A comparison with Pole Placement method shows that the sliding mode controller designed using LMI method produce a better performance responses.

ABSTRAK

Sejak bertahun lamanya, para penyelidik cuba mencari penyelesaian untuk masalah penstabilan sistem yang tak pasti. Kawalan Struktur Berubah telah dicipta untuk mengatasi masalah ini. Di dalam projek ini, Kawalan Mod Gelongsor, yang merupakan salah satu cabang dari Kawalan Struktur Berubah, telah digunakan untuk menstabilkan sistem yang tak pasti. Menggunakan cara Ketidaksamaan Matrik Datar, Matrik Permukaan Gelongsor diperolehi dan di gunakan di dalam Kawalan Mod Gelongsor yang kemudian di masukkan ke dalam model sistem Elektrohidraulik. Sistem ini kemudian di simulasikan menggunakan perisian MATLAB/SIMULINK. Perbandingan dengan cara Peletakan Pole menunjukkan Kawalan Mod Gelongsor yang di cipta menggunakan cara Ketidaksamaan Matrik Data menghasilkan reaksi system yang lebih bagus, berprestasi tinggi dan tegap.

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